Settlement Gap Risk

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Abstract

Exchanging collateral has emerged as the market standard for mitigating counterparty credit risk in the interbank derivatives market.[1] Collateral postings do not, however, eliminate that risk completely. Most notably, the so-called gap risk remains, which is the risk that in the event of counterparty default, mismatches[2] between the collateral account and the portfolio market value build up between the last margin call and the end of the close-out process[3]. Such gaps are most pronounced when the portfolio market value changes heavily during the margin period of risk. Since portfolio value changes are driven by market movements on the one hand and by incoming or outgoing contractual cashflows on the other hand, gap risk decomposes into a component driven by market movements and a component driven by settlement payments.[4] In this article, we separate these components from one another. We thereby introduce two new risks which we think should be measured and managed separately. The settlement induced component of gap risk, which we call Settlement Gap Risk (SGR), can be the dominant source of gap risk on individual time buckets, and it will not be fully covered by bilateral exchange of initial margin. We present a framework for measuring, managing and mitigating these two new risks, building upon a Monte Carlo based exposure simulation.

1 Introduction

Next to close-out netting of market values, exchanging collateral or Variation Margin (VM) is the most important mitigant of counterparty credit risk in today's interbank derivatives market. Since VM is based on

*The opinions expressed in this article are those of the authors and do not necessarily reflect the official view of d-fine GmbH or Norddeutsche Landesbank.

[1]The exchange of bilateral variation margin is even subject to recent regulatory requirements, see EBA/JC/CP/2015/002 ([1]).

[2]These mismatches may result in positive exposure towards the counterparty both in the case that the amount of collateral posted by the counterparty is insufficient and in the case that the amount of collateral posted to the counterparty is superabundant, outstanding collateral not being insolvency proof.

[3]This period is called the margin period of risk.

[4]The gap risk induced by settlement payments during the closeout period is e.g. mentioned in [2] at the end of Section 6.4.2.

the portfolio's current market value, potential changes in the portfolio's market value during the margin period of risk are not covered by VM. In an attempt to close this gap, regulators ask banks to additionally exchange Initial Margin (IM). However, IM only takes market risk induced changes of portfolio market values into account.

In this paper, we introduce a type of risk which is covered neither by VM nor by IM and which reflects the fact that large contractual cashflows may cause substantial mismatches between the portfolio's new market value and the collateral account. This component of gap risk is what we call Settlement Gap Risk (SGR). For example, SGR is created by a collateralised out-of-the-money portfolio which matures and for which outstanding collateral is not redeemed. Similarly, SGR materialises if an in-the-money portfolio moves further into the money with the counterparty not answering the resulting collateral call.

We will give an overview of SGR from a credit risk management point of view in Section 2, including means of avoiding SGR and, if no such means turn out to be implementable, ways to handle SGR in a credit risk management context. In Section 3, we give precise definitions and develop a theoretical framework for SGR, whereas in Section 4 we present numerical methods for computing SGR via Monte Carlo simulation methods and for separating it from classical gap risk exposure profiles. In Section 5, we present computational results for one of the SGR computation techniques developed in Section 4.

Let us mention that our framework also applies to the risk neutral setting. However, a discussion of the implications of SGR for the computation of valuation adjustments such as CVA/DVA/FVA lies beyond the scope of this article. We refer to [3] Section 9 for a discussion of settlement induced exposures driving CVA/DVA in the presence of Initial Margin.

2 Managing SGR

We see two possibilities for completely eliminating Settlement Gap Risk:

(i) Posting VM via a custodian instead of bilaterally: today's market standard is to post VM bilaterally. Posting VM to a custodian would eliminate SGR. VM posted to a custodian would, however, not be rehypothecable, which would create funding and liquidity issues.

(ii) Netting of cash flows and induced collateral flows: if banks were to settle contractual payments directly via collateral accounts – meaning that if a counterparty A had to pay money to counterparty B, then B would take as much of this money as possible from its collateral account, and A would post only the remaining difference – then this procedure would eliminate any delay between settlement payments and subsequent collateral redemptions, and SGR would disappear. However, such a practice would introduce higher complexity into interbank payments, and it is presently not the market standard.

Given today's market standard regarding interbank settlement processes, SGR cannot be eliminated easily and, hence, must be managed. We propose to separate SGR from replacement risk.[5] A benefit from doing so is that the remaining "purified" replacement risk profile is much better manageable: a pre deal risk analysis typically involves checking that the PFE profile of the client's portfolio, including the new trade, fits under the

[5]Replacement risk is measured by Potential Future Exposures (PFE), which are quantiles of gap exposures in the real world measure.

given replacement risk limit in every single time bucket. With SGR included in the PFE profile, this constraint leads to the following dilemma: either the limits granted would have to be be unreasonably high in order to allow for trades causing SGR; these high limits could then lead to overly high trading volumes in asset classes less susceptible to creating SGR.[6] Or, alternatively, given reasonably low limits, certain trades prone to creating SGR – like FX forwards or XCCY swaps – would need to be prohibited. Removing SGR from PFE profiles[7] provides a solution for this predicament since the "socket" PFE profile with SGR excluded is much smoother than the original PFE profile, making limit management processes more reliable: for the socket PFE profile, reasonable limits can be set, so that the pre deal limit constraints are then a valuable tool in risk management.

After removing SGR from PFE profiles – and assuming it has not been mitigated –, we face the question of how to deal with it. In our opinion, the most rigorous approach would be to treat and limit it as a new risk which then can be embedded into existing risk management processes.

An alternative approach that is currently being discussed amongst practitioners is to treat it as settlement risk, since it only appears on and immediately after future payment settlement days. This approach would, however, entail the following:

(i) First, it would increase the complexity in handling settlement risk: Settlement risk exposures are usually modelled by deterministic cashflows. Adding SGR would introduce a stochastic component.

(ii) Second, for collateralized trades, the above approach would lead to higher utilisations in settlement risk. Some trades might even exhibit settlement exposures where there has not been any exposure before at all. For example, this applies to trades whose cash flows are settled via a custodian and hence are traditionally considered as being free from settlement risk.

Thus, putting SGR into settlement risk results in higher complexity of the settlement risk framework, the implications of which should be balanced carefully.

Finally, an approach which can be observed in practice consists in simply removing the SGR which is due to future settlement payments from replacement risk, without delimiting it at all. Then only those SGR exposures are shown in replacement risk utilisations which result from realised gaps between past settlement payments and yet outstanding collateral redemptions. In this approach, replacement risk can still be managed reasonably on the socket PFE profile away from the short end of the time axis[8], while cash flow gaps become visible at the near end of the time axis once they occur and can be managed accordingly.[9]

[6] SGR is localised at settlement dates. However, since these sweep along the time axis as time goes on, a single settlement payment in the far future like it is e.g. created by a cross currency swap with terminal notional exchange would necessitate high limits on essentially all time buckets.

[7] We define SGR on a path by path level. The PFE profile with SGR removed is then defined in terms of quantiles of the SGR-free exposures.

[8] Unless an actual default event occurs, only the first two business days should be impacted by the past settlement induced SGR since collateral calls are usually answered within two business days.

[9] Let us point out that for this approach, either high limits must be granted for the near time buckets, which may not be desirable, or occasional SGR induced limit breaches on those buckets must be expected.

3 Theoretical framework

In this section, we give precise definitions for SGR, and we explain how to separate SGR from classical gap risk.

Given a scenario ω for the future, the exposure at a future time t between an institution and a defaulting counterparty with close-out terminating at time t given ω for a given netting set is given by

$$E_\omega(t) = \quad [V_\omega(t) - C_\omega(t')]^+ \ , \qquad (1)$$

where

(i) t' is the time of the last collateral payment before t, given that a default event occurred with close-out terminating at t,

(ii) V is the market value process of the netting set,

(iii) C is the collateral balance process and

(iv) we set $[\cdot]^+ = \max(0, \cdot)$.

This formula reflects the assumption that both counterparties fulfil their contractual payment obligations (excluding collateral payments) during the close-out period. Although this supposition is still most commonly used for MPR modelling, it is currently subject to discussions (see [3] 3.2). We would like to point out that our framework does not depend on this assumption, in the sense that it carries over verbatim to other MPR models.[10]

If the collateral is given by $C_\omega(t') = V_\omega(t-\Delta)$, then

$$E_\omega(t) \ = \ [V_\omega(t) - V_\omega(t-\Delta)]^+ \ , \qquad (2)$$

[10] Indeed, the exposure socket which we will define below is independent from any choice of MPR model, while the SGR part, defined as the difference between the classical exposure and the exposure socket, will clearly depend on how classical exposure is modelled.

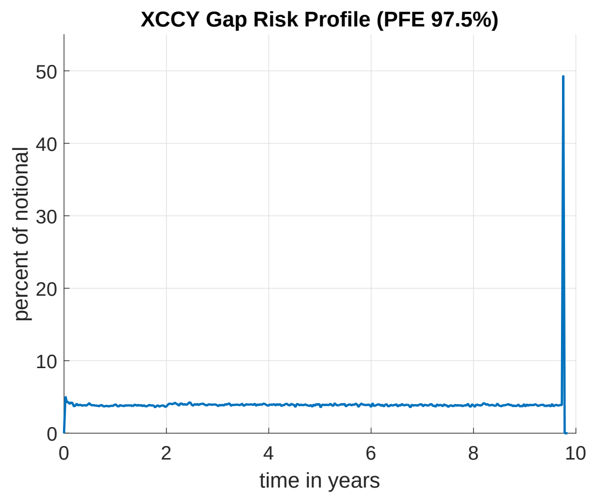

Figure 1: Gap risk profile for a collateralized cross currency swap with terminal notional exchange payment

and $E_\omega(t)$ is then proportional to a difference quotient of $V_\omega(t)$ floored at zero. Hence, market value jumps induced by contractual payments result in spikes for $E_\omega(t)$. These narrow but marked spikes, which represent SGR, may outweigh the exposure away from settlement dates by several orders of magnitude; see Figure 1.

In this chapter, we discuss methods for writing the overall exposure, in each given scenario, as a sum

$$E_\omega(t) \ = \ E_\omega^{\text{socket}}(t) \ + \ E_\omega^{\text{SGR}}(t) \ , \qquad (3)$$

with E^{SGR} reflecting the SGR part of the exposure, that is, the exposure gap which is due to settlement payments.

We consider a single netting set of trades between an institution and a counterparty, partially covered by a single collateral agreement; the case of multiple CSAs within a netting set is a straightforward generalisation. We assume that we are given a model consisting of

(i) a probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ describing the possible future histories of the world and their probabilities,

(ii) a filtration $(\mathcal{F}_t)_{t\geq 0}$ of $\mathcal{F}$ such that for each $t \geq 0$, $\mathcal{F}_t$ represents the information available at time t,

(iii) a margin period of risk function $\Delta(t)$ such that $t - \Delta(t)$ is the last margin call date before t given that the counterparty defaults with close-out terminating at time t,[11]

(iv) an adapted process V describing the future market value of the netting set,

(v) an adapted process V^{coll} describing the future market value of the collateral set, and

(vi) an adapted process C describing the future collateral balance.

Then the exposure process is given by

$$E(t) \;=\; [V(t) - C(t - \Delta(t))]^+ \;, \qquad (4)$$

where we set $C(t) = C_0$ for $t \leq 0$, with C_0 denoting the initial collateral balance. We further assume that for each $\omega \in \Omega$ and for each margin call date $t > 0$, the collateral account value $C_\omega(t)$ depends only on the path $(V_\omega(t'))_{0\leq t'\leq t}$ and on CSA parameters like e.g. a threshold amount. That is, we assume that for each t, there exists a function $\Gamma_t : \mathbb{R} \times \mathbb{R}^{[0,t]} \to \mathbb{R}$ such that for each $\omega \in \Omega$,

$$C_\omega(t) \;=\; \Gamma_t(C_0, V^{\text{coll}}_\omega|_{[0,t]}) \;. \qquad (5)$$

We refer to [2] Section 6.4.2 for a detailed discussion of how to model collateral within a Monte Carlo simulation.

3.1 Exposure decomposition

Settlement Gap Risk is generated by settlement flows during the close out period. In order to define the complementary gap risk socket, that is, the market movement induced component of gap risk, we define a modified collateral process which, to determine the collateral at time t, ignores certain settlement payments in the margin period of risk $[t, t+\Delta(t)]$. To define this modified collateral process, let us assume that for any time interval δ, we are given a modified adapted value process V^*_δ which, in a sense which we will make precise below, ignores certain cashflows in that time interval[12]. Let us write $V^{\text{coll},*}_\delta$ for its restriction to the collateralized part.[13] We then define C^*_δ via

$$C^*_{\delta,\omega}(t) \;:=\; \Gamma_t(C_0, V^{\text{coll},*}_{\delta,\omega}|_{[0,t]}) \qquad (6)$$

for $t > 0$ and $C^*_\delta(t) := C_0$ for $t \leq 0$. We then define the resulting socket exposure process by setting

$$\begin{aligned} &E^{\text{socket},*}(t) \\ &= \left[V(t) - C^*_{[t-\Delta(t),t]}(t - \Delta(t))\right]^+ \;. \end{aligned} \qquad (7)$$

That is, to define the exposure socket, we consider a modified collateral balance which ignores certain cash flows during the margin period of risk.[14] The corresponding SGR process is then defined by

$$E^{\text{SGR},*} \;:=\; E - E^{\text{socket},*} \;. \qquad (8)$$

In this definition of $E^{\text{SGR},*}$ we do not split the initial collateral balance C_0 in order to single out the settlement gap risk induced by settlements which have occurred in the past. Thus, the SGR induced by past settlements is

[11] In particular, $\Delta(t)$ is bounded below by the counterparty's assumed margin period of risk. We are focusing on positive exposures.

[12] We will eventually only consider the case where $\delta := [t - \Delta(t), t]$ is equal to the margin period of risk ending at time t. Let us point out that in particular we allow δ to depend on the time parameter t.

[13] We assume that the mapping $V \mapsto V_\delta$ is additive in V.

[14] Our definition of the exposure socket should be compared to [3] 3.2 (6), particularly in the live cash flow methodology case.

still part of the exposure socket. The resulting practical implications have been discussed at the end of Section 2.

Let us moreover point out that $E^{\mathrm{SGR},*}$ can be negative, with negative values reflecting exposure reducing effects of payments effected by the defaulting counterparty within the margin period of risk. One could also define SGR as $(E - E^{\mathrm{socket},*})^+$, thereby obtaining an additive decomposition of a more conservative exposure $E^{\mathrm{socket},*} + (E - E^{\mathrm{socket},*})^+$ which does not assume the defaulting counterparty to fulfil its contractual payment obligations within the closeout period; cf. the discussion in [3] 3.2 and also footnote 10.

3.2 Live cashflows

Regarding reasonable definitions of V_δ^*, let us recall that in an arbitrage free and complete market, for a pricing measure $\mathbb{Q}$ and a corresponding numeraire instrument N, the market value of the netting set at a future time t is given by

$$V(t) \;=\; N(t) \cdot \sum_{t' \geq t} \mathbb{E}^{\mathbb{Q}} \left[\frac{F_{t'}}{N(t')} \middle| \mathcal{F}_t \right] \;, \qquad (9)$$

where $F_{t'}$ is the stochastic future flow paid by the netted deals at date t'.[15] To obtain the δ-adjusted value process with respect to the live cashflow methodology, we set

$$V_\delta^{\mathrm{lc}}(t) \;:=\; N(t) \cdot \sum_{t' \geq t \,,\, t' \notin \delta} \mathbb{E}^{\mathbb{Q}} \left[\frac{F_{t'}}{N(t')} \middle| \mathcal{F}_t \right] \;, \qquad (10)$$

and we call the resulting modified exposure process $E^{\mathrm{socket,lc}}$ the exposure socket with respect to the live cashflow methodology. It describes the exposure one would be facing if the last future margin call before close-out was made on the assumption that all settlement flows during the margin period of risk were withheld.

[15] Here an American option is regarded as a Bermudan option with daily exercise rights.

3.3 Live trades

As for alternative definitions of V_δ^*, let us assume that the netting set decomposes into components[16] α with value processes V_α and – possibly path dependent – maturities t_α, where we require that $V_\alpha(t) = 0$ for $t > t_\alpha$. Setting

$$V_\delta^{\mathrm{lt}}(t) \;:=\; \sum_{t_\alpha \geq \max(\delta)} V_\alpha(t) \;, \qquad (11)$$

we obtain a resulting modified exposure process $E^{\mathrm{socket,lt}}$ which we call the exposure socket with respect to the live trade methodology. It describes the exposure one would be facing if the last future margin call before close-out was made on the assumption that all terminal settlement flows at trade maturities during the margin period of risk were withheld. The corresponding SGR process $E^{\mathrm{SGR,lt}}$ contains precisely the exposure caused by terminal settlement payments at trade maturities.

Let us note that the live trade methodology depends on the chosen decomposition of the netting set as well as on the chosen notion of trade maturity.[17] Let us also point out that the live cashflow methodology can be recovered as a special case of the live trade methodology by considering, for each future date, the fictive trade consisting of all stochastic flows in the netting set which are due on that date.

4 Numerical methods

Without loss of generality, we consider a single netting set N together with a collateral agreement which covers a part $N_{\mathrm{coll}} \subseteq N$ of

[16] We call such a component a trade, even though it does not need to be an economically or legally defined actual trade.

[17] For instance, there are two canonical notions of trade maturity for a cash settled Bermudan swaption, a path dependent one and a path independent one.

that netting set. We assume that N is presented to us as a collection of constituents $\alpha \in N$ which may be trade packages, trades or single cash flows.

We furthermore assume that we are building upon an existing exposure simulation engine which uses a predefined exposure aggregation time grid $\{t_1, \ldots, t_n\}$[18] and which, for each constituent $\alpha \in N$, provides a future value grid $(V_{\alpha,i,j})_{i,j}$ above that time grid, where i is a path index and where j is a time index; we set $V = \sum_{\alpha \in N} V_\alpha$ and $V_{\text{coll}} = \sum_{\alpha \in N_{\text{coll}}} V_\alpha$.

We moreover assume that the implementation provides a margin period of risk function Δ on $\{t_1, \ldots, t_n\}$ as well as a collateral grid $(C_{i,j})_{i,j}$, where $C_{i,j}$ models the amount of collateral posted for N_{coll} at time $t_j - \Delta(t_j)$ on path ω_i. That is, in terms of the notation of Section 3,

$$C_{i,j} \;=\; C_{\omega_i}(t_j - \Delta(t_j))\ . \tag{12}$$

The pathwise future exposure is then given by

$$E_{\omega_i}(t_j) \;=\; [V_{i,j} - C_{i,j}]^+\ .$$

In the following, we discuss algorithms providing modified collateral grids $C^*_{i,j}$ which yield implementations of the live trade methodology and the live cashflow methodology respectively.

4.1 Live trades

To describe an implementation of the live trade methodology, we will assume that for each time index j, there is a function $\Gamma_j : \mathbb{R} \times \mathbb{R}^j \to \mathbb{R}$ such that

$$C_{i,j} \;=\; \Gamma_j(C_0, V_{\text{coll},i,1}, \ldots, V_{\text{coll},i,j})\ . \tag{13}$$

This covers the case where simulated collateral is obtained via linear interpolation on the market value grid.

[18]The simulation may be performed on a grid different from the exposure aggregation grid.

Remark 1 We assume that equations (5) and (13) are compatible in the sense that Γ_j uses $V_{\text{coll},i,k}$ with $t_k > t_j - \Delta(t_j)$ merely in order to obtain a reasonable interpolated value for $V_{\text{coll},\omega_i}(t_j - \Delta(t_j))$.

The functions Γ_j^{lin} can easily be modified in a way so that they take a threshold amount into account. Another example is given by a Brownian Bridge interpolation method.[19]

Definition 1. *We define the live trade grid C^{lt} by setting*

$$C^{\text{lt}}_{i,j} \;:=\; \Gamma_j(C_0, V^{\text{lt},j}_{\text{coll},i,1}, \ldots, V^{\text{lt},j}_{\text{coll},i,j})\ .$$

where

$$V^{\text{lt},j}_{\text{coll},i,k} \;:=\; \sum_{\alpha \in N_{\text{coll}} \text{ still alive at } t_j} V_{\alpha,i,k}\ .$$

That is, only constituents which are still live at time t_j are taken into account for determining the collateral to be used for computing the exposure at time t_j.

Remark 2 Let us note:

(i) The live trade methodology algorithm described above depends strongly on the decomposition of the netting set. In particular, if trades are single cash flows, then it gives an implementation of the live cashflow methodology.[20]

(ii) The above methodology also depends on a reasonable definition on when a constituent is still live. One may use pathwise or path independent notions of maturity; the former are to be preferred e.g. to capture the SGR generated by cash

[19]To estimate the volatilities to be used in a Brownian Bridge setup, multiple paths may need to be considered. We ignore this additional complexity in our notation since it is not relevant for our discussion.

[20]In practice though, managing a value grid V_α for every single cashflow α will often be unfeasible in terms of memory requirements.

settled Bermudan swaptions. One may also choose to consider a trade to be essentially matured if its market value is exactly zero.

4.2 Live cashflows

To describe an algorithm implementing the live cashflow methodology, we will make the following assumptions:

(i) We assume that the margin period of risk function $\Delta : \{t_1, \ldots, t_n\} \to \mathbb{R}$ satisfies the property that the map $t_j \mapsto t_j - \Delta(t_j)$ is injective, so that t_j can be recovered from $t_j - \Delta(t_j)$. We moreover assume that the latter map is order preserving.[21]

(ii) We assume that $(C_{i,j})_{i,j}$ is determined by simulating market values on the time grid $\{t_{j_0} - \Delta(t_{j_0}), \ldots, t_n - \Delta(t_n)\}$, where t_{j_0} is the smallest element of $\{t_0, \ldots, t_n\}$ such that $t_{j_0} - \tau > t_0$. That is, we assume that

$$C_{i,j} := \begin{cases} \hat{V}_{\mathrm{coll},i,j} & \text{if } j \geq j_0 \\ C_0 & \text{otherwise} \end{cases} ,$$

where we write $\hat{V}_{\mathrm{coll}i,j}$ to denote is the simulated market value of N_{coll} on path ω_i at time $t_j - \Delta(t_j)$.[22] Let us note that this assumption is compatible with equation (5), while it is different from equation (13).

(iii) For each $\alpha \in N_{\mathrm{coll}}$, each path index i and each time $j \geq j_0$, we write $\hat{V}^{\mathrm{lc}}_{\alpha,i,j}$ for the time $t_j - \Delta(t_j)$ market value of the derivative which has exactly the same cash flows as α, except that it has no payments in $[t_j - \Delta(t_j), t_j]$. We assume that we are able to compute these modified market values on every path.[23]

Given these assumptions and writing

$$\begin{aligned} \tau &= \{t_0, \ldots, t_n\} \\ \tau' &= \{t_{j_0} - \Delta(t_{j_0}), \ldots, t_n - \Delta(t_n)\} \end{aligned} ,$$

the live cashflow methodology can be implemented as follows:

(i) Perform the risk factor diffusion on the time grid

$$\tau'' = \tau \cup \tau' .$$

(ii) When the simulation reaches a point $t_j - \Delta(t_j) \in \tau'$, then for each trade $\alpha \in N_{\mathrm{coll}}$ and for each path index i, compute $\hat{V}^{\mathrm{lc}}_{\alpha,i,j}$.

(iii) If the simulation reaches a point $t_j \in \tau$, then for each trade $\alpha \in N_{\mathrm{coll}}$ and for each path index i, compute $V_{\alpha,i,j}$.

(iv) If the simulation reaches $t_j \in \tau' \cap \tau$, then both steps (i) and (ii) above need to be carried out.

(v) Define

$$C^{\mathrm{lc}}_{i,j} := \begin{cases} \hat{V}^{\mathrm{lc}}_{\mathrm{coll},i,j} & \text{if } j \geq j_0 \\ C_0 & \text{otherwise} \end{cases} . \qquad (14)$$

Again, this formula is easily modified to allow for nontrivial threshold amounts.

Remark 3 The above algorithm can be generalized to the case where the map

$$t_j \mapsto t_j - \Delta(t_j)$$

[21] These assumptions are e.g. satisfied if Δ is constant. As we explain in Remark 3, they can be dropped, at the cost of slightly complicating the implementation.

[22] Obvious modifications to this formula can be made in order to take threshold amounts into account.

[23] This is clearly the case for products which are technically represented as linear combinations of cash flows whose payment dates are deterministic: for these, at time $t - \tau$, simply all flows falling in the time interval $[t - \tau, t]$ must be ignored for present value computations.

is not necessarily injective or order preserving. Indeed, let J denote the set of $j \in \{0, \ldots, n\}$ such that $t_j - \Delta(t_j) > t_0$, and let us redefine $\tau' := \{t_j - \Delta(t_j)\,;\, j \in J\}$. Then if the simulation reaches a point $t' \in \tau'$, we consider the set $J' \subseteq J$ of all j such that $t_j - \Delta(t_j) = t'$, and we compute $\hat{V}^{\mathrm{lc}}_{\alpha,i,j}$ for all $j \in J'$.

4.3 Summary

Let us summarize the pros and cons of the above algorithms:

(i) The implementations of both the live trade methodology and the live cashflow methodology are rather cheap in terms of computational costs and memory requirements.[24]

(ii) The live cashflow methodology is harder to implement since separate time and market value grids need to be managed. Additional complexity of this kind would be introduced if path dependencies were to be taken into account for collateral simulation.[25] Moreover, the live cashflow algorithm may be difficult to implement for products which are not represented as linear superpositions of cash flows with deterministic payment dates.

(iii) The live trade methodology is easy to implement, without requiring additional grids and without further assumptions on the portfolio's constituents. Moreover, it easily takes path dependent collateral representations into account. However, it only captures terminal payments, and its implementation depends on a reasonable notion of trade maturity.

[24] This holds except when the live trade algorithm is (ab)used to provide an implementation of the live cashflow methodology. In general, the cost scales linearly with the number of constituents of the netting set.

[25] Such path dependencies are e.g. introduced by accurate treatments of minimal transfer amounts.

5 Computational results

We implemented the live cashflow methodology, building upon the pricing and exposure engine *MoCo* which is a proprietary development of d-fine. For the computational results shown below, we used a hybrid Hull-White-FX model on a simulation and exposure aggregation time grid featuring a mesh size of 10 calendar days. We assumed that the margin period of risk has a length of 10 calendar days and that all CSAs provide daily margining. For examples 1, 2 and 3, we took the threshold amount to be zero, while for example 4, we assumed a threshold of one million euros. The profiles are based on 5000 paths.

Example 1 (Collateralized IRS) Figure 2 shows the PFE profile of a collateralized ATM interest rate swap in EUR. The corresponding socket exposure quantiles obtained via the live cashflow methodology are shown in Figure 3, while the corresponding SGR quantiles are shown in Figure 4. We see that the market risk induced exposure socket dominates the near future and linearly goes to zero over time, while the influence of SGR is approximately constant over time. In particular, the spikes clearly dominate the far future. This is easily explained by the fact that a single SGR spike only sees a single cashflow over the margin period of risk, while the exposure socket sees all future payments and their potential present value changes over the margin period of risk.

Example 2 (Collateralized XCCY forward) Figures 5, 6 and 7 show the corresponding profiles for a collateralized ATM cross currency floating-floating swap with terminal notional exchange. We see that the SGR caused

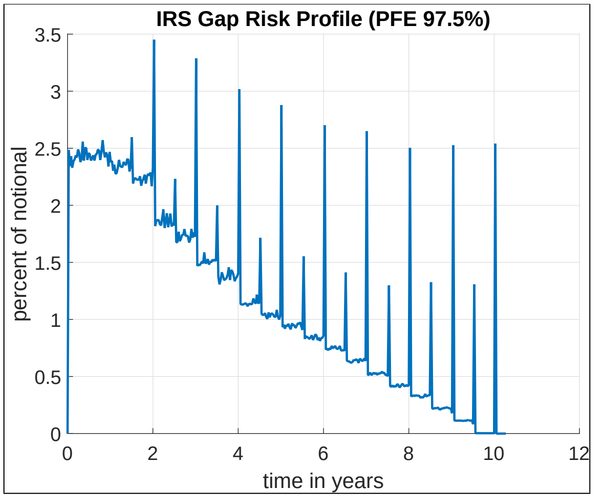

Figure 2: Gap risk profile of a collateralized plain vanilla swap (including SGR)

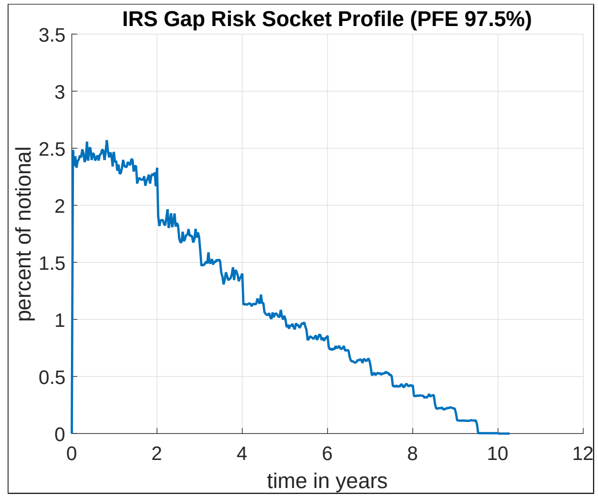

Figure 3: Gap risk socket profile of a collateralized plain vanilla swap (excluding SGR)

by the final notional exchange clearly dominates the overall picture.

Example 3 (Collateralized real world portfolio) Figures 8, 9 and 10 show the corresponding profiles for a collateralized portfolio obtained by randomly perturbing an actual real life portfolio containing interest rate swaps, XCCY swaps and physically settled European swaptions in three currencies. We see that again SGR spikes dominate the exposure profiles.

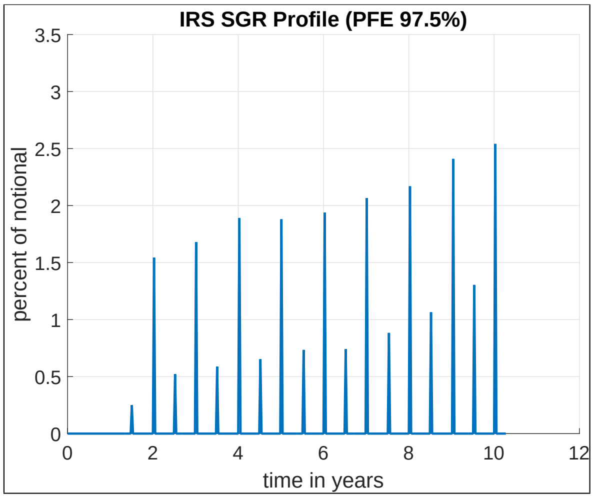

Figure 4: SGR profile of a collateralized plain vanilla swap

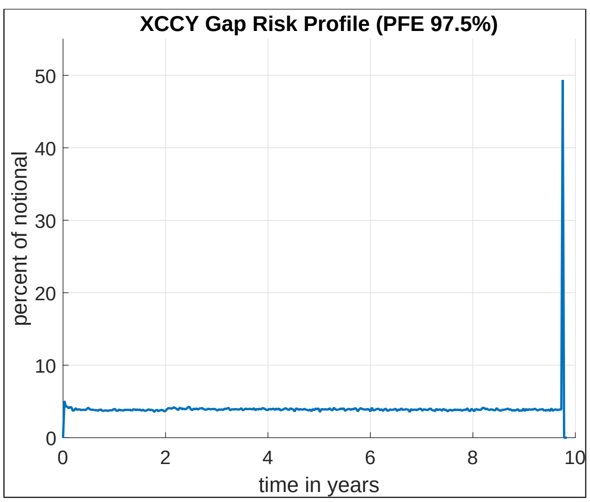

Figure 5: Gap risk profile of a collateralized cross currency swap (including SGR)

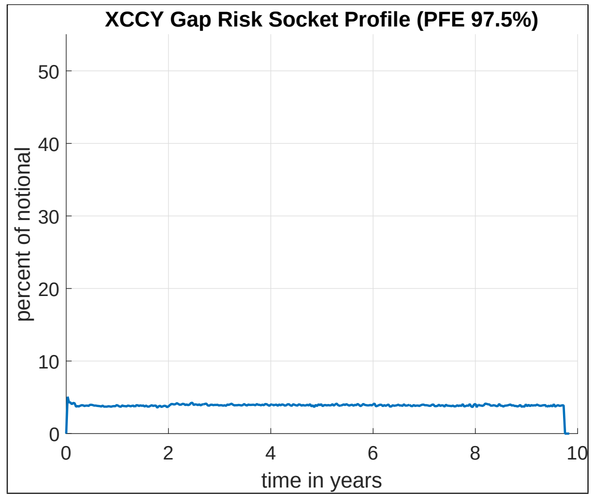

Figure 6: Gap risk socket profile of a collateralized cross currency swap (excluding SGR)

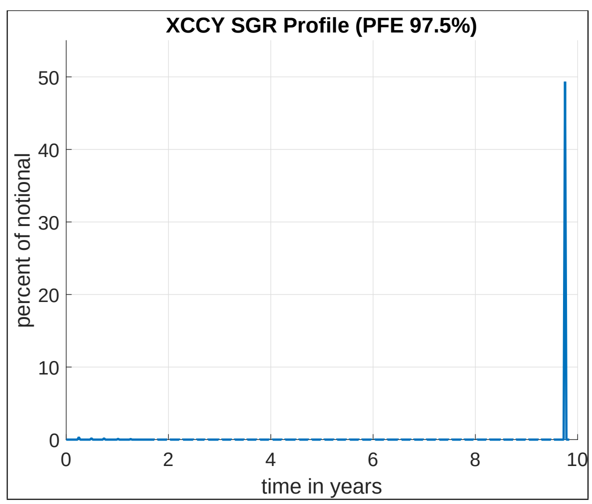

Figure 7: SGR profile of a collateralized cross currency swap

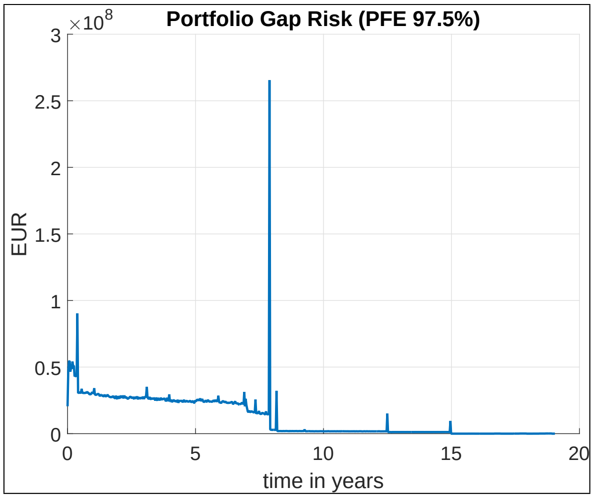

Figure 8: Gap risk profile of a collateralized real world portfolio (including SGR)

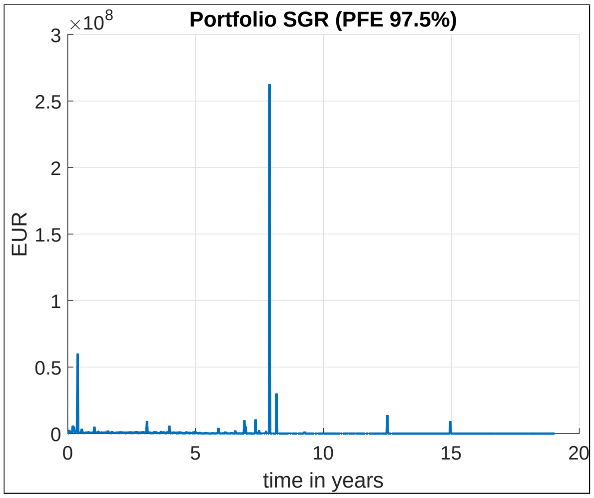

Figure 10: SGR profile of a collateralized real world portfolio

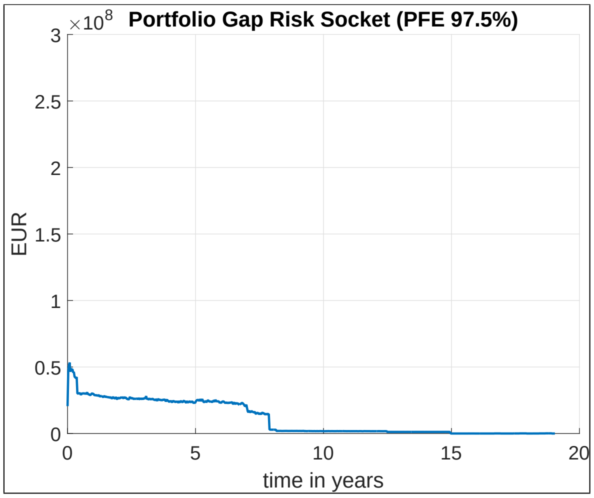

Figure 9: Gap risk socket profile of a collateralized real world portfolio (excluding SGR)

6 Conclusion

We have introduced SGR as a new type of risk which is present whenever variation margin is exchanged between counterparts trading OTC derivatives. We have pointed out that SGR is not being mitigated by exchanging initial margin, in contrast to the ”classic” part of gap risk which is induced by changes of market parameters. We have proposed different ways of mitigating SGR, for separating SGR from classical gap risk and for managing SGR, providing both a theoretical framework and numerical methods. Finally, we have presented numerical results based on the live cashflow methodology.

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