# Computing MVA via regression and principal component analysis

Christian Kappen*

July 12, 2017

*d-fine GmbH, Frankfurt, Germany*

**Abstract**

MVA is today's price of the future costs generated by future initial margin postings. Computing MVA requires long-term risk neutral simulations of future initial margin amounts. The ISDA SIMM[1] (cf. [1]) computes initial margin based on the portfolio's sensitivity with respect to a high-dimensional vector of risk factors.[2] In this note, we describe a way to approximate future SIMM based initial margin amounts in terms of regression functions with respect to a small number of explanatory variables. Our method uses principal components analysis, and it fits in naturally with American Monte Carlo techniques.

**Keywords:** MVA, initial margin, ISDA SIMM, AMC

## 1 Introduction

According to [2], posting initial margin for non-cleared OTC derivatives has become mandatory for financial institutions with more than 3€ trillion notional in OTC derivatives starting September 2016, and more institutions will be phased in until September 2020. ISDA has proposed an initial margin methodology which is becoming a market standard, cf. [1]. In order to assess the costs of future initial margin postings (MVA), financial institutions will need to perform long-term risk neutral simulations of future ISDA SIMM based initial margin amounts.

For simulating future price scenarios, it is common practice to use American Monte Carlo techniques, where prices or continuation values are described via regression functions in so-called explanatory variables; see [8] for one of the pioneering articles in this direction, and see [4] Chapter 8.6 for a comprehensive treatment of the subject. Once such regression functions have been determined, they can be differentiated; forward sensitivities with respect to explanatory variables can thus be obtained at essentially no cost, provided that the simulation has full rank in the explanatory variable space in the sense explained in Section 2. In this note, we explain how forward sensitivities in explanatory variables can be used to approximate future ISDA SIMM initial margin amounts. Our method uses principal components analysis, and it fits in naturally with American Monte Carlo techniques.

Concretely, we propose to

- embed the explanatory variable space into the ISDA SIMM risk factor space in a model compatible way, for each relevant future point in time, such that both the price function gradient and the main ISDA SIMM eigenmodes are largely tangent to the explanatory variable space and then

*Correspondence to: d-fine GmbH, An der Hauptwache 7, D-60313 Frankfurt, Germany. E-mail: christian.kappen@d-fine.de

[1] Standard Initial Margin Method; it can be regarded as the present day market standard for assessing initial margin in the uncleared OTC derivatives business

[2] ISDA SIMM considers 60 interest rate risk factors per currency.

- apply the ISDA SIMM formulae to the (computable) projection of the price function gradient to the explanatory variable space.

The error resulting from this approximation scales linearly in the projection of the price function gradient to the orthogonal parts of the ISDA SIMM eigenmodes[3], weighted by their eigenvalues; see Proposition 3.7 for a precise statement. Let us point out that the method of first mapping ISDA SIMM risk factors to explanatory variables in a model compatible way and then applying the chain rule to obtain sensitivities does not necessarily provide a viable method for computing or approximating MVA, as can be seen from Remark 3.5.

The methodology proposed in the note will be implemented and further analysed as part of an MSc thesis which the author is currently working on.

# 2 Forward sensitivities with respect to explanatory variables

In this preliminary section, we recall the fact that in order to be able to compute sensitivities with respect to explanatory variables by differentiating regression functions, the simulated samples in the explanatory variables space must not be contained in a strictly smaller dimensional subspace of that space.

Indeed, if that condition was violated, then the problem of fitting regression functions would become non-unique, and partial derivatives with respect to explanatory variables would be ill-defined.

For example, let us consider the polynomial function $f$ in the two variables $X$ and $Y$ which is given by $f(X,Y) = X \cdot Y$, and let us assume that all sample points $(x_i, y_i)$ lie on the diagonal subspace defined by $X = Y$. Then regressing polynomials in $\mathbb{R}[X,Y]$ to the points $(x_i, y_i, f(x_i, y_i))$, the regression functions $g_1(X,Y) = X^2$, $g_2(X,Y) = Y^2$ and $g_3(X,Y) = X \cdot Y$ provide equally good fits; the $g_i$ with $i \in \{1,2,3\}$ even yield the same results in all points of the diagonal space. However, the partial $X$-derivatives $\partial_X g_i$ are completely different, even on the diagonal, and the same holds for the partial $Y$-derivatives $\partial_Y g_i$. Intuitively speaking, one can say that in order to compute $(\partial_X, \partial_Y)$, one needs to be able to vary $X$ while keeping $Y$ fixed and vice versa, which is clearly not possible of all samples are confined to the diagonal space where $X$ moves along with $Y$.

As a corollary to this observation, we see that if a simulation is based on a one factor interest rate model, then it is not possible to use regression functions in order to simulate the two-dimensional partial derivatives vector of future prices of a European swaption with respect to the swap rate and the annuity. Indeed, in the one-dimensional subspace traced by the simulation, the annuity will move with the swap rate, in the sense that when the swap rate goes up, which corresponds to a high rate scenario, then the annuity will go down, so it is not possibly to vary the two independently from one another within the simulation.

We also see that if we tried to use all ISDA SIMM risk factors as explanatory variables in order to fit regression functions for computing forward sensitivities with respect to these risk factors, then an unreasonably high-dimensional simulation would need to be used.[4]

# 3 Forward sensitivities with respect to general risk factors

In the following, we assume that the rank criterion given in Section 2 is satisfied so that forward sensitivities with respect to the given explanatory variables are well-defined. Let us point out that in order to ensure that this condition is satisfied even at the start of the simulation, early start Monte Carlo techniques must be used, as had been suggested in [5] and as they have been developed by Wang

[3] If $v_i$ is a normalized ISDA eigenmode, then by its orthogonal part we mean its orthogonal component with respect to the given embedding of the explanatory variable space

[4] In [5], Green and Kenyon propose to augment the dimensionality of the simulation state space by overlaying simulated scenarios with real world shocks. However, as Green and Kenyon point out, it is only possible to obtain prices in these shocked scenarios via closed-form pricing formulae: pricing via regression only works when scenarios are generated consistently in the risk neutral measure.

and Caflisch in [9]. We now consider the problem of approximating forward sensitivities with respect to risk factors different from the explanatory variables.

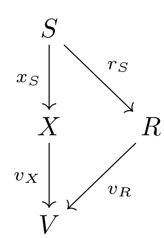
## 3.1 General outline of the problem

To phrase this problem in a general setting, let us consider, for a fixed future point in time, the state space of the simulation $S$, the space $X$ of explanatory variables relevant for the financial derivative under consideration, the space $R$ of risk factors considered within the context of ISDA SIMM, and the present value space $V = \mathbb{R}$, where the unit length on $V$ is given by a unit of currency. We allow $S$, $X$ and $R$ to be general Riemannian manifolds; for our local purposes, however, we may assume that they are open subsets of finite dimensional real Euclidean spaces. We refer to [7] and to [6] for generalities regarding smooth and Riemannian manifolds respectively.

**Example 3.1** *In order to illustrate the abstract framework outlined in this section, it may be useful to consider the following basic example: at the simulation date $t_0$, a one factor Hull White interest rate model driving a single rate curve in a single currency is calibrated to the market, and we are interested in simulating the ISDA SIMM based initial margin amount at a future time $t > t_0$ for a forward starting swap with start date $> t$. In this case, we may choose $S$, $X$, $R$ and $V$ as follows*[5]*:*

| | | | |
|---|---|---|---|
| $S$ | $:=$ | $\mathbb{R}$ | space of possible short rates at time $t$ |
| $X$ | $:=$ | $\mathbb{R}$ | space of possible forward swap rates at time $t$ with respect to the given swap |
| $R$ | $:=$ | $\mathbb{R}^{12}$ | space of continuously compounded zero rates at the standard tenors, at time $t$ |
| $V$ | $:=$ | $\mathbb{R}$ | space of possible time $t$ values of the given swap |

A simulation state $s \in S$ implies a value of the explanatory variables and a value of the ISDA SIMM risk factors. Moreover, a value of the explanatory variables implies a price, and so does a value of the ISDA SIMM risk factors. For practical purposes, we may assume that the corresponding maps $x_S : S \to X$, $r_S : S \to R$, $v_X : X \to V$ and $v_R : R \to V$ are smooth[6]. Intuitively speaking, $R$ is the full state space, while $S$ is a simplified state space which corresponds to the simulation model.The situation is summarised by the following diagram:




It should be noted that the maps $x_S$ and $r_S$ can be assumed to be known or implemented algorithmically and that the map $v_X$ is given in terms of the regression coefficients. The map $v_R$, while it is known to exist in theory, may not be given explicitly in practice. We may and will assume that the outer diagram commutes, i.e. that $v_R \circ r_S = v_X \circ x_S$. This consistency condition means that the price associated to a model state coincides with the price attached to the risk factor state which is attached to a model state.

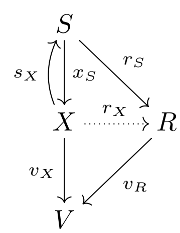
**Example 3.2** *In the context of Example 3.1 above, we define the maps $x_S$, $r_S$, $v_X$ and $v_R$ using the bond reconstruction formula for the Hull-White model which allows to obtain simulated yield curves from simulated short rates, cf. [3] Proposition 10.1.7. More precisely speaking:*

[5] The ISDA weights are specified in the coordinate frame on $R$ which is given by the fair swap rates at the standard tenors. It is possible to pass from this frame to the one given by continuously compounded zero rates via a change of coordinates.

[6] A map between Riemannian manifolds is called smooth if it is infinitely differentiable in all points of its domain.

| | |
|---|---|
| $x_S$ | maps a time $t$ short rate to the corresponding time $t$ forward swap rate implied by the model via the bond reconstruction formula |
| $r_S$ | maps a time $t$ short rate to the corresponding time $t$ zero rates at the standard tenors implied by the model via the bond reconstruction formula; the one-dimensional image $\mathrm{im}(r_S) \subseteq R$ is exactly the space of yield curve scenarios predictable by the given Hull-White one factor model |
| $v_X$ | maps a time $t$ forward swap rate to the corresponding time $t$ value of the swap implied by the model; at least implicitly, this map recovers the short rate from the forward swap rate and then determines the corresponding annuity via the bond reconstruction formula |
| $v_R$ | maps a time $t$ yield curve configuration to the corresponding time $t$ value of the swap; this map is model independent (up to the choice of an interpolation methodology for the yield curve) |

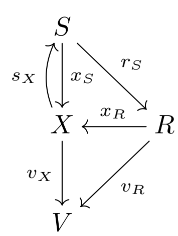
As we already pointed out, the space $R$ will be high-dimensional, while $S$ and $X$ will be low-dimensional. The rank condition described in Section 2 can formally be stated by requiring $x_S$ to be a submersion[7]. After shrinking around a point of interest $s \in S$, we may assume that $x_S$ has a section $s_X : X \to S$ such that $S$ is a trivial bundle over $X$[8]. In many cases, the map $r_S$ will be a smooth embedding, but we do not require this to be the case. We obtain the following diagram:




(1)

The section $s_X$ is not unique, and the composite map $r_X := r_S \circ s_X$, which will often turn out to be a smooth embedding, is not unique either. However, the resulting lower triangle commutes for any choice of $s_X$. Indeed, $v_R \circ r_X = v_R \circ r_S \circ s_X = v_X \circ x_S \circ s_X = v_X$.

**Example 3.3** *In the setting of Example 3.1 and Example 3.2 above, we may assume that $x_S$ is bijective and that, hence, the section $s_X = x_S^{-1}$ is unique.*[9] *Let now $x_R : R \to X$ be the natural map sending a yield curve scenario to the corresponding forward swap rate:*




*Then the composite map $v_X \circ x_R$, while it coincides with $v_R$ on the image of $r_S$ by construction, does not coincide with $v_R$ on all of $R$. Indeed, for a yield curve scenario $r \in \mathrm{im}(r_S)$, there is a yield curve scenario $r' \in R$ with the same associated forward swap rate $x_R(r') = x_R(r)$ but with a different associated annuity. For such an $r'$, we have*

$$v_R(r') \neq v_R(r) = v_X(x_R(r)) .$$

[7] This amounts to saying that the differential $dx_S$ of $x_S$ induces surjections of tangent spaces $(dx_S)_s : T_sS \to T_{x_S(s)}X$ for all $s \in S$.

[8] The map $s_X$ being a section of $x_S$ means that $x_S \circ s_X$ is the identity on $X$.

[9] This holds locally around a point $s \in S$ and its $x_S$-image in $X$.

*In particular, one* cannot *use the chain rule to obtain the differential $dv_R$ from the differentials $dv_X$ and $dx_R$. Intuitively speaking, the derivative of $v_R$ in the direction of changing the annuity while keeping the swap rate fixed is not captured by $v_X \circ x_R$. This would be particularly problematic if instead of the price of a forward starting swap we would be considering the price of the tradable asset which is given by the annuity.*

**Example 3.4** *If, in the context of Example 3.3, the short rate $r$ itself was used as an explanatory variable, one could have the idea to consider, on $\mathrm{im}(r_S)$ and for a given tenor $i$, the short rate $r$ as a function of $r_i$ by inverting the bond reconstruction formula for $i$. The resulting map $x_{R,i} : \mathrm{im}(r_S) \to X$ would result in a commutative lower triangle; that is, $v_R|_{\mathrm{im}(r_S)} = v_X \circ x_{R,i}$. Taking its derivative and using the chain rule would, however, not yield the partial derivative of $v_R$ with respect to $r_i$: indeed, when the short rate moves with $r_i$, then so do the $r_j$ with $j \neq i$.*

In particular, we obtain:

**Remark 3.5** *As Examples 3.3 and 3.4 show, the differential $dv_R$ cannot be computed by writing the explanatory variables as functions of the ISDA SIMM risk factors in model compatible way and applying the chain rule afterwards.*

The differential $dv_R$ thus remains as elusive as the map $v_R$ itself. This is a serious problem, given that $dv_R$ is needed as an input to the ISDA SIMM formulas in order to simulate future initial margin amounts.[10]

We do not know how to compute $dv_R$ without having some explicit knowledge about $v_R$, e.g. when $v_R$ is given explicitly via analytical pricing formulas. However, we are going to explain how to approximate the ISDA SIMM initial margin amount associated with $dv_R$ in terms of the derivatives of the explicitly given maps in diagram (1).

## 3.2 Recasting of the ISDA SIMM formulae

We recast the ISDA SIMM formulae for the interest rate delta part of initial margin for a single curve in a single currency[11] using the geometric framework outlined above. For the original form of these formulae, we refer to [1] Section B no. 8.

In this case, the space $R$ is identified with $\mathbb{R}^n$ for $n = 12$ via the coordinate system which is given by the fair swap rate $r_i$ at the ISDA standard tenors, measured in basis points. The differentials $dr_i$ then provide relative coordinates of the cotangent bundle $T^*R$ of $R$, and the corresponding dual basis, written $\{\partial_{r_i}\,;\, 0 \leq i \leq n\}$, is a natural coordinate system of the tangent bundle $TR$. We define the constant tangent vector $w \in TR$ via

$$w(dr_i) \;=\; 1 \;\forall 1 \leq i \leq n$$

or, equivalently, via

$$w = \sum_{i=1}^{n} \partial_{r_i} \;.$$

ISDA specifies a constant positive definite symmetric bilinear form $Q$ on $T^*R$ via

$$Q(dr_i \otimes dr_j) \;=\; \rho_{ij} \cdot \mathrm{RW}_i \cdot \mathrm{RW}_j \quad \text{with } 1 \leq i, j \leq n \;,$$

[10] Before the advent of MVA, the map $v_R$ and its derivative $dv_R$ were not needed, because the model price associated to a model state $s \in S$ is given by $v_R \circ r_S = v_X \circ x_S$, which is due to the commutativity of the outer square in diagram (1).

[11] In this note, we focus exclusively on the interest rate delta part of the ISDA SIMM formulas. Restricting attention to a single curve does not induce any loss of generality. The argument can easily be extended to several currencies, cf. Remark 3.6.

where the $\mathrm{RW}_i$ and the $\rho_{ij}$ are the risk weights and the symmetric positive definite tenor correlation matrix specified by ISDA respecitvely.[12] Now first the concentration risk factor CR is defined via

$$\mathrm{CR} \;=\; \max\left(1, \frac{w(dv_R)}{T}\right) \;,$$

where $T \in \mathbb{R}_{>0}$ is a scalar threshold amount specified by ISDA. Let us point out that while

$$w(dv_R) \;=\; \sum_i \partial_{r_i} v_R$$

is a smooth $\mathbb{R}$-valued function on $R$, the function CR, while being continuous, is not necessarily smooth. Now ISDA defines the interest rate delta initial margin to be

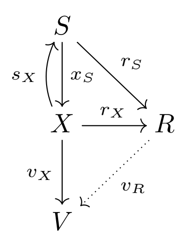
$$\mathrm{IM} \;=\; K \;=\; \mathrm{CR} \cdot \sqrt{(Q(dv_R \otimes dv_R)} \;.$$

Again, while $\sqrt{(Q(dv_R \otimes dv_R)}$ is a smooth $\mathbb{R}$-valued function on $R$, the continuous $\mathbb{R}$-valued function $K$ on $R$ is not necessarily smooth.

**Remark 3.6** *For multiple curves and multiple currencies, ISDA takes additional inter-curve and inter-corrency correlation factors into account. Our methods presented in Section 3.3 carry over, except that in addition to $w(dv_R)$ and $Q(dv_R \otimes dv_R)$, the smooth function $w'(dv_R)$ must be approximated, where $w'$ is the tangent field defined by $w'(dr_i) = \mathrm{RW}_i$.*

## 3.3 Approximating initial margin

We now explain how to approximate the ISDA SIMM interest rate delta initial margin amount IM defined in Section 3.2 above. Let us first recall the commutative diagram (1)




where we have now drown all explicitly known and computable maps as solid lines and the unknown map $v_R$ as a dashed line.

### 3.3.1 Formulation in terms of the total differential

Since the lower triangle commutes, the pullback of $dv_R$ to $T^*X$ is equal to $dv_X$. We now endow $R$ with the standard Euclidean metric. Let us assume that $r_X$ is an immersion, and let us endow $X$ with the Riemannian metric induced by $r_X$. Then the pullback tangent space $r_X^*(TR)$ naturally decomposes as a direct sum $TX \oplus (TX)^\perp$, and this decomposition induces a dual decomposition of cotangent spaces which is compatible with the natural pairing between tangent and cotangent spaces and which is compatible with the projection of cotangent spaces induced by $r_X$. Letting $w^\parallel \in TX$ and $w^\perp \in (TX)^\perp$ denote the two components of $w$ with respect to this decomposition of $r_X^*(TR)$, we obtain, for any point $x \in X \subseteq R$,

$$dv_R(w)_x \;=\; dv_X(w^\parallel)_x \,+\, dv_R(w^\perp)_x \;.$$

[12] The risk weights and the correlations are calibrated by ISDA to partially stressed historical data. Risk weights are measured in basis points.

Here the first summand is computable, and we regard the second summand as an incomputable error term.

Similarly, letting $\lambda_1 \geq \cdots \geq \lambda_n$ denote the eigenvalues[13] of $Q$ and letting $v_1, \ldots, v_n$ denote a set of corresponding normalised eigenvectors with $X$-parallel components $v_i^{\|}$ and $X$-orthogonal components $v_i^{\perp}$, we obtain an analogous statement. Let us phrase it as a proposition, for future reference:

**Proposition 3.7** *The form $Q$ satisfies the equation*

$$Q(dv_R \otimes dv_R)_x \;=\; \sum_{i=1}^{n} \lambda_i \cdot dv_R(v_i)_x^2 \;=\; \sum_{i=1}^{n} \lambda_i \cdot (dv_X(v_i^{\|})_x + dv_R(v_i^{\perp})_x)^2$$

*at any $x \in X \subseteq R$. This quantity can be written as the sum of the computable quantity*

$$\sum_{i=1}^{n} \lambda_i \cdot dv_X(v_i^{\|})_x^2 \;,$$

*and the incomputable error term given by*

$$\sum_{i=1}^{n} \lambda_i \cdot (2 \cdot dv_X(v_i^{\|})_x \cdot dv_R(v_i^{\perp})_x + dv_R(v_i^{\perp})_x^2) \;.$$

**Remark 3.8** *The computable restriction $dv_X$ of $dv_R$ is given naturally, and it is independent of any choice of metric on $R$. In order to project the ISDA SIMM vectors to $X \subseteq R$, one could try using a metric which is different from the standard Euclidean metric.*

### 3.3.2 Dual formulation in terms of the gradient

We can recast the above method by using the gradient $\nabla v_R \in TR$ instead of the total differential $dv_R \in T^*R$, as follows: Let $\langle \cdot, \cdot \rangle$ denote the Euclidean scalar product on $TR$; then $\nabla v_R$ is defined via $\langle \nabla v_R, \nu \rangle \;=\; dv_R(\nu)$ for all $\nu \in TR$; that is, $\nabla v_R$ is the Euclidean dual of $dv_R$. Now the restriction of $\nabla v_R$ to $X \subseteq R$ decomposes as a sum

$$\nabla v_R|_X \;=\; (\nabla v_R)^{\|} \;+\; (\nabla v_R)^{\perp} \;,$$

where the computable quantity $(\nabla v_R)^{\|} \;=\; \nabla v_X \in TX$ is tangent to $X$ and where the unknown quantity $(\nabla v_R)^{\perp} \in r_X^*(TR)$ is orthogonal to $X$. Hence, for any $x \in X$, we have an induced decomposition

$$dv_R(w) \;=\; \langle \nabla v_R, w \rangle \;=\; \langle \nabla v_X, w^{\|} \rangle \;+\; \langle (\nabla v_R)^{\perp}, w^{\perp} \rangle \;,$$

and similarly

$$Q(dv_R \otimes dv_R) \;=\; \sum_{i=1}^{n} \lambda_i \cdot \Big( \langle \nabla v_X, v_i^{\|} \rangle \;+\; \langle (\nabla v_R)^{\perp}, v_i^{\perp} \rangle \Big)^2 \;.$$

Intuitively speaking, the restriction $v_X$ of $v_R$ captures the initial margin contribution of the sensitivities along $X$, while it ignores the sensitivities perpendicular to $X$. The neglected sensitivities orthogonal to $X$ enter $Q(dv_R \otimes dv_R)$ only via their projections to the eigenmodes of $Q$, weighed by the respective eigenvalues. Hence, even if the error which is induced by passing from $v_R$ to $v_X$ can be non-negligible when regarding the sensitivities themselves, it may turn out to be of marginal importance for initial margin if the main ISDA SIMM eigenmodes are largely parallel to $X$.

[13] These eigenvalues are measured in basis points to the power of two.

**Example 3.9** *In the context of Examples 3.1, 3.2 and 3.3 above, we may simplistically say that the given one factor short rate model captures parallel movements of the yield curve. Let us assume that for a given simulated point of the state space, the first eigenmode of $Q$ is exactly the model induced yield curve movement, with an eigenvalue of*

$$\lambda_1 = 27000\,\mathrm{bp}^2\ ,$$

*and that the third eigenmode of $Q$ is a skew movement with an eigenvalue of*

$$\lambda_3 = 2400\,\mathrm{bp}^2\ .$$

*Let us moreover consider an interest rate derivative which, at the given simulated point of the state space, is sensitive exactly to the first and the third eigenmode of $Q$, such the the derivative's value changes by 50 units of currency under a parallel shift of $1\,\mathrm{bp}$ and by 50 units of currency under a skew by $1\,\mathrm{bp}$. Then passing from $v_R$ to $v_X$ amounts to disregarding the considerable skew sensitivity. However, the comparatively small ISDA SIMM eigenvalue of the skew eigenmode results in a more acceptable overall error for $Q(dv_R \otimes dv_R)$: indeed, the approximate value is*

$$\sqrt{27000\,\mathrm{bp}^2 \cdot (50\,\text{units of currency per basis point})^2} \;=\; 8216\,\text{units of currency}\ ,$$

*while the error term, on the other hand, is*

$$(\sqrt{2400 \cdot 50 \cdot 50})\,\text{units of currency} \;=\; 2449\,\text{units of currency}\ .$$

*So neglecting concentration risk factors, the relative error for the future ISDA SIMM initial margin obtain by our approximation lies around 23%, even though we neglected 'half' of the relevant sensitivities.*

All in all, we obtain the intuitively plausible result that an explanatory variable space $X$ together with a model compatible embedding $r_X : X \hookrightarrow R$ can be expected to yield good approximations of future initial margin amounts if it is sufficiently tangent to the concentration risk vector $w$, to the main ISDA eigenmodes and to the gradient of the true price function $v_R$.

An analysis of the ISDA correlations and risk factors (see [1] Section D) shows that the most material ISDA eigenmodes in the swap par rate coordinate frame are given by the parallel shift, the tilt and the skew; cf. Table 1.

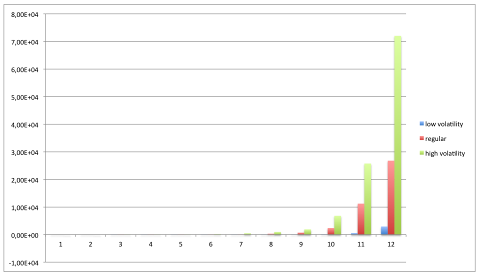
## 3.4 Suggestions for practical applications

Regarding practical applications of the approximation method described in this note, we suggest the following:

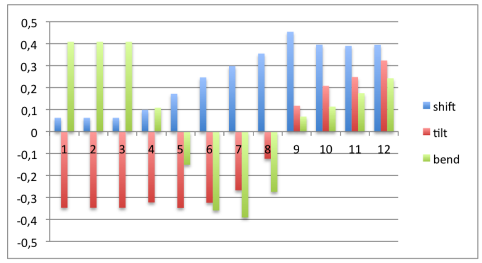
1. Use the ratio of the simulation start day's approximate initial margin amount[14] and the start day's true ISDA SIMM initial margin amount in order to scale all simulated future approximate initial margin amounts.
2. For a collection of sufficiently generic (and possibly synthetic) derivatives and on a possibly coarse sub-grid of the simulation time grid, occasionally compute the maps $v_R$ and their gradients explicitly, via nested Monte Carlo simulations if necessary, in order to assess the error of the approximation and in order to thus ensure the applicability of this note's methodology for a given model, a given choice of explanatory variables and a given portfolio.
3. If necessary and feasible[15], augment the dimensionality of the explanatory variable space $X$ or the simulation state space $S$ in order to reduce the approximation error.

[14] In order to apply this paper's method to the start date of the simulation, early start Monte Carlo techniques must be used, as was already pointed out at the beginning of Section 3.

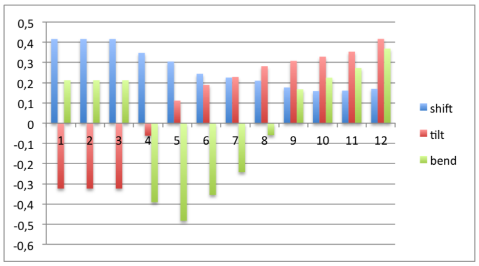
[15] Feasibility is i. a. contingent to the availability of sufficiently many sufficiently liquid calibration instruments.



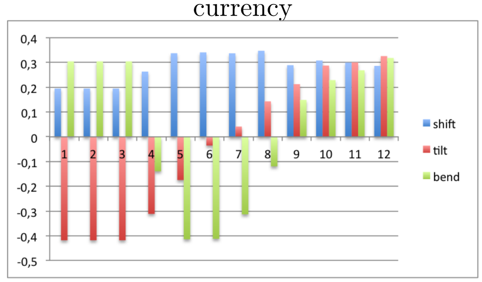

Eigenvalues




First three eigenmodes for low volatility currency




First three eigenmodes for regular volatility currency




First three eigenmodes for high volatility currency

Table 1: ISDA quadratic forms eigendecompositions

# 4 Summary

We explained how to approximate future initial margin amounts and, hence, MVA, using a commonplace low-dimensional Monte Carlo simulation. Our technique is based on a principal components analysis (PCA) of the quadratic form given by the ISDA risk weights and correlations, and it fits in naturally with American Monte Carlo methods.

# References

[1] ISDA SIMM - Methodology, version R1.3, Effective Date: April 1, 2017 (`https://www2.isda.org/attachment/OTIzMQ==/ISDA%20SIMM%20vR1.3%20(PUBLIC).pdf`).

[2] Margin requirements for non-centrally cleared derivatives (`www.bis.org/bcbs/publ/d317.htm`).

[3] Leif B. G. Andersen and Vladimir V. Piterbarg. *Interest Rate Modeling*. Atlantic Financial Press, 2010.

[4] Paul Glasserman. *Monte Carlo Methods in Financial Engeneering*. Springer, 2003.

[5] Andrew Green and Chris Kenyon. MVA: Initial Margin Valuation Adjustment by Replication and Regression. *Risk*, 28(5), 2015.

[6] John M. Lee. *Riemannian manifolds: an introduction to curvature*. Graduate texts in mathematics. Springer, New York, NY [u.a.], 1997.

[7] John M. Lee. *Introduction to smooth manifolds*. Graduate texts in mathematics. Springer, New York, NY [u.a.], 2. ed. edition, 2012.

[8] Francis A. Longstaf and Eduardo S. Schwartz. Valuing American Options by Simulation: A Simple Least-Squares Approach. *The Review of Financial Studies*, 14(1):113–147, 2001.

[9] Yang Wang and Russel Caflisch. Pricing and Hedging American-Style Options: A Simple Simulation-Based Approach. *The Journal of Computational Finance*, 13(4):95–125, Summer 2010.