Quantum Deep Hedging: Potential of quantum-based machine learning methods as a hedging strategy

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The key technology of machine learning / artificial intelligence (AI) has undergone a perceptible revolution in recent years, developing into an important strategic, economic and industrial success factor. The use of AI methods for classification, anomaly identification and regression analysis is increasingly significant in the financial sector, for example in terms of portfolio optimisation strategies. In this context, we will be presenting our analyses and results on utilisation of what is known as "quantum deep hedging", which describes hedging of financial instruments based on deep neural networks (DNNs) with "reinforcement learning" techniques and elements of quantum neural networks.

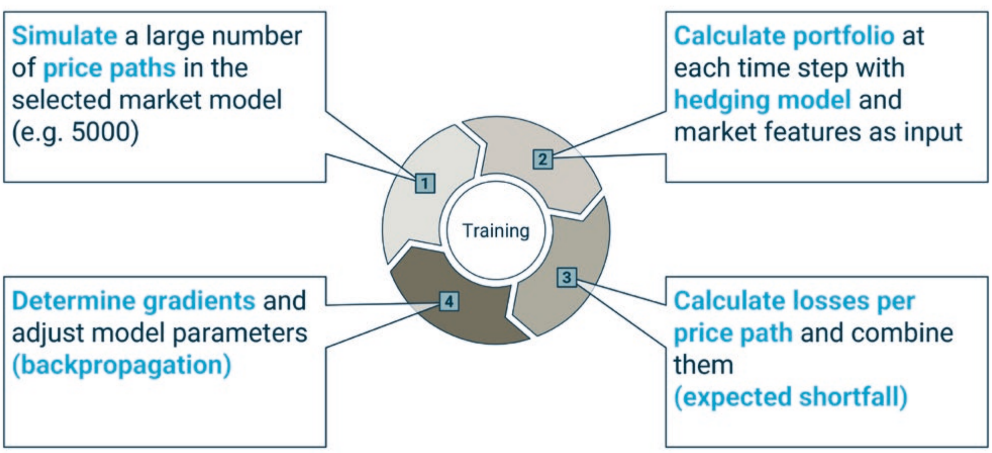
Hedging

In general, hedging refers to the strategy of mitigating against, reducing or minimising risks that can occur due to negative price changes in the trading of financial instruments. Common standardised hedging methods include delta or gamma hedging, as well as Monte Carlo simulation based methods. Developing a successful hedging strategy is extremely challenging. As well as the natural dynamics of the market and the numerous correlations between different instruments, influences from secondary conditions such as liquidity, limitation of capital and transaction costs add to the complexity of the problem.

Reinforcement learning

Machine learning systems can essentially be classified based on particular criteria, such as the type of supervision. Alongside supervised or unsupervised learning, what is known as "reinforcement learning" represents a separate methodology, in which the learning system (also known as the "agent") interacts with its environment and takes actions based on certain sets of rules, for which it receives either a positive or negative reward (or penalty). The rules are updated according to the learning steps until an optimum strategy has been identified. The aim of the agent is to learn a behaviour that maximises its expected rewards.

Fig. 01: Schematic diagram of a deep hedging model training process.



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Deep hedging

Deep hedging involves modelling trading decisions within the hedging strategy using a deep neural network, enabling different additional factors such as past trading decisions, liquidity restrictions or news analysis to be integrated into the overall model as "features". During calibration of a stochastic pricing model that acts as an input for the neural network, either the implicit volatility method or a classic time series method based on historic value development can be used. The aim of the neural network learning process is to minimise a defined mathematical objective function within the formulation of a mathematical optimisation problem. In the case examined here, we use the 10% expected shortfall as the risk measure.

The basic relevant steps in training the neural network are shown in ▶ Fig. 01. In the technical implementation, the "pfhedge" framework [1], which is based on the Pytorch library [2], was used and developed [4]. The particular purpose of the framework is to provide a comprehensive model for market processes (e.g. Black-Scholes, Heston, Merton jump) and instruments (e.g. European, binary or look back options).

Fig. 02: Illustration of the deep neural network (DNN) including the input/output layer and various hidden layers (left). Detailed breakdown of the hidden layers showing the neurons (in this case a "perceptron") as components of the different layers. Graphic adapted from [3].

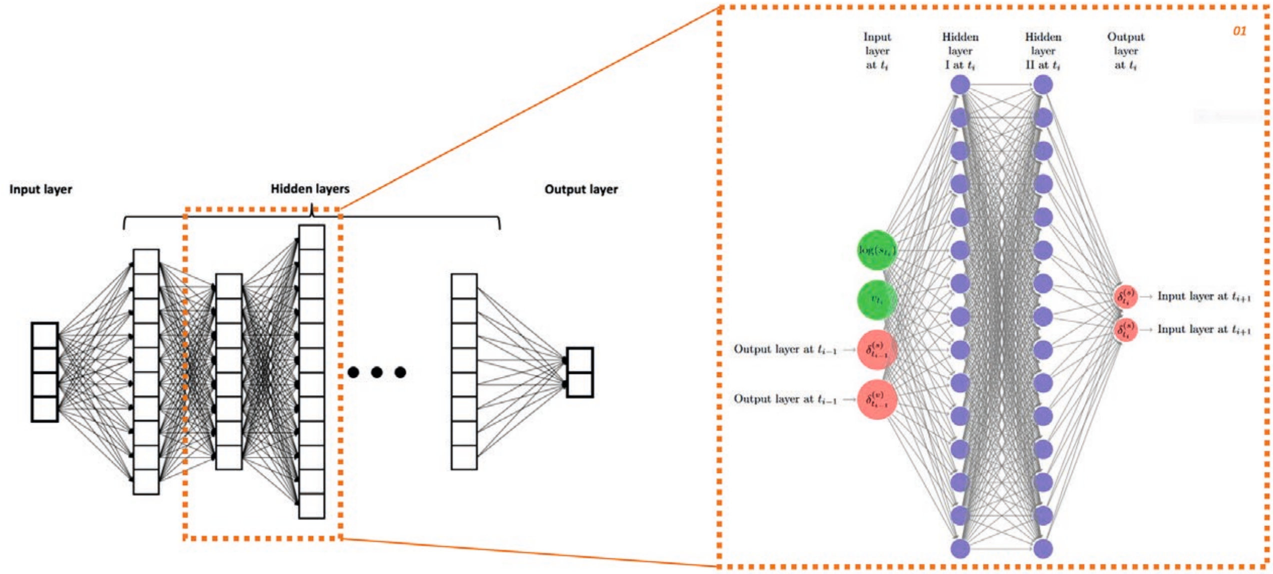

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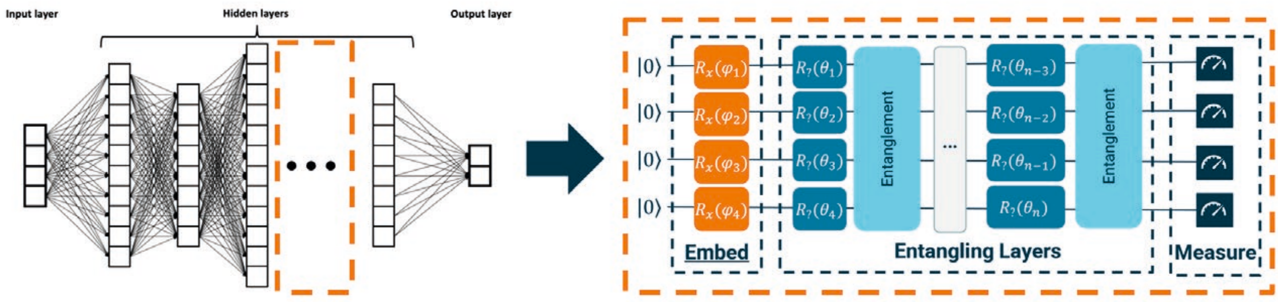
Fig. 03: Hybrid architecture of the deep neural network with quantum components. A hybrid neural network is created by replacing parts of classic hidden layers with trainable quantum components.



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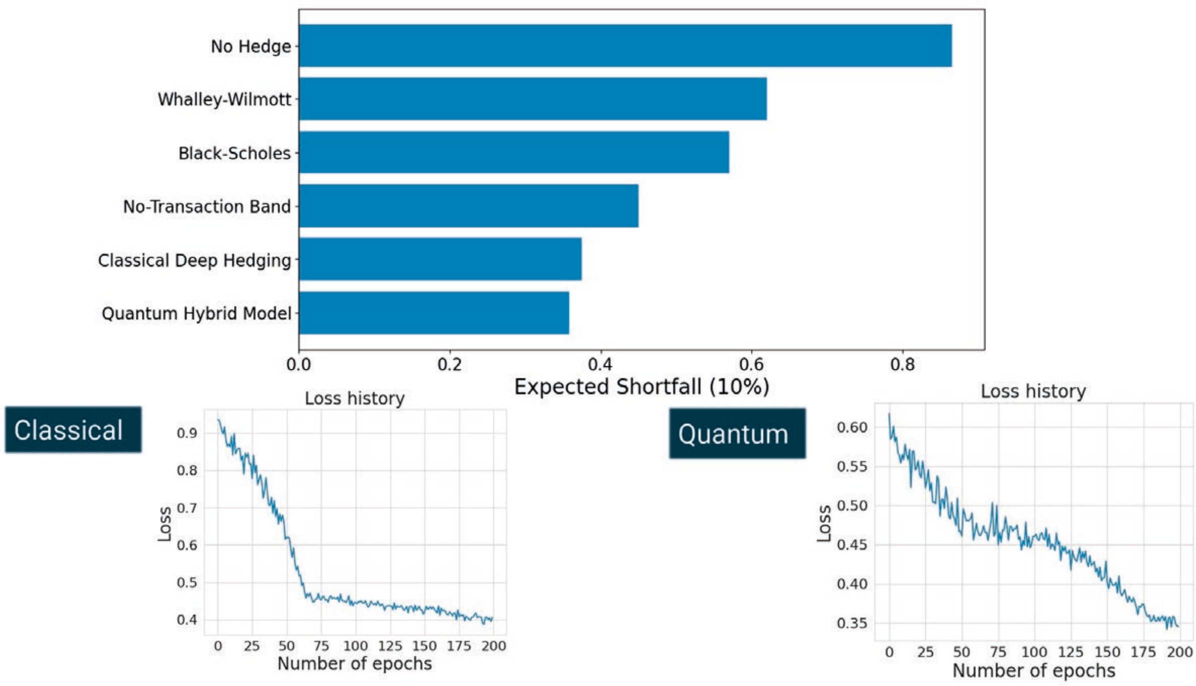
Quantum computing

The first basic thoughts about designing a quantum computer emerged several decades ago now. However, technical realisation of different architectures has only really gained significant momentum in recent years, mainly due to development by major technology companies. The hoped-for quantum advantage over classic computers is based on three fundamental properties of quantum systems and quantum states. These are superposition, entanglement of quantum states and the possibility of interference between states.

In classic computers, the traditional bit with binary states b={0,1} represents the smallest unit of information content. The analogue in a quantum computer is the qubit, which is represented by a quantum-mechanical state projected as one of the two basic states 0 or 1 only at the time of physical measurement. Without the measurement process, the quantum state is generally in a superposition of the two basic states, in other words it can be simultaneously 0 and 1. With the presence of n qubits, this results in exponential scaling of the possible basic states to 2^n. Qubits are manipulated using (unitary) operators (similar to classic logic gates), which can be combined into quantum gates by iterative execution and switching. The parameters of these operators (for example rotation by a particular angle) can be seen as a kind of analogy for the weights and biases of the neurons in an artificial neural network, creating

Fig. 04: Comparison of classical and quantum hybrid models in a restrictive setting (limited number of parameters and training epochs).
Top: The quantum hybrid model performs comparably to classic deep hedging and significantly better than traditional methods. Bottom: Progression of respective loss functions over the training epochs.



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the relationship to classic machine learning. Although many of the hoped-for quantum advantages are only likely to manifest themselves with more sophisticated, error-adjusted future hardware, the method is suitable for the currently available hardware with the errors it contains.

Quantum deep hedging

While classic deep hedging has been able to achieve significant improvements over explicit hedging models and is therefore already used in practice in isolated cases, studying the possible advantages of expanding to quantum neural networks (QNNs) is the subject of cutting edge research. In particular, the focus is on the possibilities of dynamic adaptations in an environment with market changes, the fact that explicit assumptions regarding the underlying asset types are not necessary, and the option of taking into account extreme events. ▶ Fig. 03 shows the introduction of a quantum component in a neural network by replacing a classic layer with a quantum circuit. One advantage of the selected approach is the comparatively low susceptibility to quantum noise.

The training of classical layers and quantum layers takes place simultaneously as shown in ▶ Fig. 01.

Results

During development, numerous different market models and parameter configurations were tried out. The results shown in ▶ Fig. 04 were simulated using a Merton jump model with large jumps and a look back option as a derivative, because this case benefits particularly well from the deep hedging approach.

In our analysis, in which hybrid models (as shown in ▶ Fig. 03) were used, a comparable performance to that of the pure classical case was achieved, even though the magnitudes that can currently be simulated did not show any significant advantage from the integration of quantum circuit components. However, there were indications that the advantages of the quantum components could manifest themselves in problems of larger dimensions. The data from ▶ Fig. 04 for small model sizes provides particularly promising indications of this. Independently of our project, similar results were achieved in [5] with different methods, demonstrating the robustness of these advantages.

One barrier to scaling in the method studied is the very large number of circuit executions required. Any efficiency increases, for example through better batching support, will allow the execution and comparison of problems with larger dimensions in the future.

Literature

[1] pfhedge Package, Code reference on Github: https://github.com/pfnet-research/pfhedge
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