New volatility conventions in negative interest environment

Current developments and necessary adjustments of IT systems in trading, risk management and accounting

1. Introduction

During the week from 10 to 14 December 2012, the supply of interest-rate volatilities (cap/floor and swaption volatilities) by the leading provider ICAP was repeatedly modified.

Providers take interest volatility data off the market

At the beginning of the week, it became apparent that the provision of so-called lognormal cap volatilities, in particular for the most important currencies for the German-speaking regions, EUR and CHF, would be discontinued. This means no less than the termination of one of the most important conventions in the interest rate derivatives market, used for decades by practically all marketable banking systems and valuation models. Furthermore, customers were told that, going forward, swaption volatilities would be available in the "Shifted Lognormal" convention as well as cap volatilities in the "Shifted Lognormal" and "Normal" conventions, along with cap and floor prices. However, on 14th December, these changes were partially reversed and the provision of lognormal data partially resumed.

Background: negative interest

For almost one and a half years now, banks and other financial market players have been observing the increasing emergence of negative interest, first with regard to the Swiss franc (as a consequence of the SNB foreign exchange policy to fix the EUR/CHF exchange rate at CHF 1.20 : EUR 1.00 as its lower limit). This effect has now also reached the euro, which is particularly evident when taking into consideration the implied forward rates (in addition to the published benchmarks), which are in turn used for the valuation of caps and floors as well as swaptions. By way of example, the following graph illustrates the negative interest on CHF for CHF LIBOR for terms up to two-months.

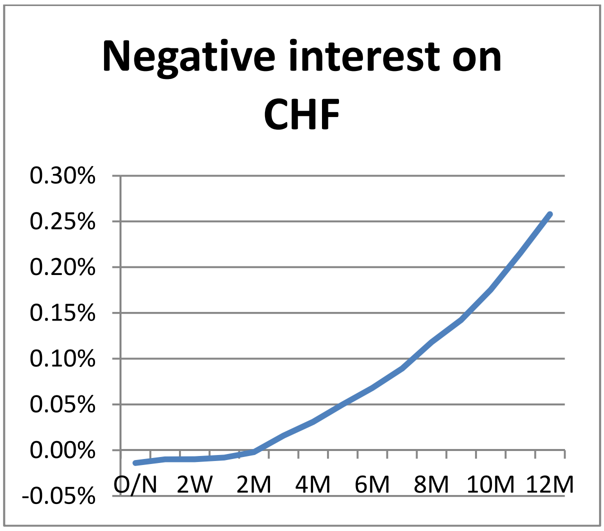

Figure 1: CHF LIBOR data of 14/12/2012

Negative (forward) interest rates have a dramatic impact on the Black'76 model used as a market standard for valuing all vanilla interest-rate options. This valuation formula breaks down technically as it contains terms such as $\log(F/K)$ that are only defined – with a positive strike rate K – for positive forward rates F. This is hardly surprising, as the Black model operates on the core assumption that the underlying value is positive.

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As a result, current negative interest rates require modified models as new standards, which should nevertheless be as simple as possible.

2. The new conventions

The data now being supplied either in addition to or as a replacement for lognormal volatilities are based on other interest rate models and thus require modified valuation procedures. Apart from the stated discounting type (collateralised vs. uncollateralised), only directly quoted option prices are free of model assumptions.

The new models and the valuation formulae for vanillas are outlined below.

Normal distribution model

Assuming the forward rate F_t can be modeled by:

$$dF_t = \sigma_N dW_t$$

W_t represents the Brownian motion and σ_N the volatility of the interest rate. The solution is given by:

$$F_t = F_0 + \sigma_N W_t.$$

It follows directly that the volatility σ_N determines the absolute size of the fluctuation around the starting value, in contrast to the lognormal model where the volatility influences the relative deflection. Within a normal model, plain vanilla options also possess a closed-form analytical solution, cf. [4]. Taking the derivative of the cumulative normal distribution

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

we have the following formulae:

$$c_N(T,K) = e^{-rT}\left[(F_t - K)N(d) + \sigma\sqrt{t}N'(d)\right]$$

for plain vanilla calls and, by way of analogy, for put options

$$p_N(T,K) = e^{-rT}\left[(K - F_t)N(-d) + \sigma_N\sqrt{t}N'(d)\right]$$

Where by: $$d = \frac{F_t - K}{\sigma_N\sqrt{t}}.$$

The advantage of the normal model compared to the displaced diffusion model, discussed later, is that it does not need any further parameters, such as shift, which may require dynamic adjustments. However, it also predicts, almost surely, arbitrarily negative interest rate phases. As a result, the use of this model is largely limited to the translation between option prices and volatilities, but it does not supply any realistic dynamics of the interest rate development. This is in contrast to the related Hull White model used, which additionally presumes a mean reversion.

Shifted lognormal model (displaced diffusion)

Reference [2] and [3] (chapter 16) contain comprehensive descriptions and analysis of the displaced diffusion model. This model is always dependent on the shift size Θ, supplying a lower limit for the values for F_t permitted by the model. Θ must either be selected beforehand or supplied by the market data provider together with the volatility. In order to reflect the possibility of negative interest, Θ<0 must be selected accordingly. The intrinsic dynamic is described by:

$$dF_t = d(F_t - \Theta) = \sigma_{DD}(F_t - \Theta)dW_t$$

giving solution:

$$F_t = \Theta + (F_0 - \Theta)\exp\left(\sigma_{DD}W_t - \frac{1}{2}\sigma_{DD}^2 t\right).$$

The familiar lognormal model (Black'76) is contained therein as the special case Θ=0. The value of an option is usually the result of the discounted empiric value of the relevant option's pay-off, under the shifted lognormal model. This leads to the following formula for a vanilla call:

$$C_{DD}(T,K) = df_T E\left[(F_T - K)_+\right] = df_T E\left[((F_T - \Theta) - (K - \Theta))_+\right].$$

As $F_t - \Theta$ is a "conventional" lognormal process, the present value of the option may be calculated using Black’76 with shifted input data:

$$C_{DD}(T,K,F_t) = C_{B76}\left(T, K-\Theta, F_t-\Theta, \sigma_{impl}^{DD}(T, K-\Theta)\right)$$

for call options and, by way of analogy,

$$P_{DD}(T,K,F_t) = P_{B76}\left(T, K-\Theta, F_t-\Theta, \sigma_{impl}^{DD}(T, K-\Theta)\right)$$

for puts.

Two points should be noted: first, for any given call (or put) price, the implicit volatilities of the displacement diffusion model and the lognormal model are not identical when the shift is not equal to zero, even in the case of a positive forward rate, i.e. the model would require a specific calibration for given option prices. The second point is closely related to the first: negative shifts Θ<0 may be associated with restrictions regarding the existence and unambiguousness of an implicit volatility within the displaced diffusion model (cf. [2], no. 3.1).

Comparison of characteristics

Upon conclusion of this theoretical section, the characteristics of all models discussed are compared in a tabled form below:

Category	Lognor-mal	Normal	Shifted LN
Interest rate	F>0	-∞ <F<∞	F>Θ (Θ<0)
Option price C/P	Black‘76	own formula	Shifted Black‘76
Volatility level	independent of interest rate	dependent on interest rate	independent of interest rate
Degree of reality	high until 2011, now partly unac-ceptable	unrealistically even deflec-tions up and down	realistic, but dynamic shift adjustments

We will now analyse the implementation options for these new models and market data conventions within the existing infrastructure of financial institutions.

3. The implications

First of all, one might ask why the problem of negative interest rates has not yet forced all market participants to migrate to other methodologies and systems, given the fact that they have impacted on market data for over a year now (cf. [1]). In fact, many institutions have already made some, albeit small, adjustments to have low positive minimum values prior to the data import of the volatility values into bank’s systems. On the other hand, many software solutions are designed for robustness and include implicit (in most cases technically incorrect) fall-back solutions for input data which are not permissible in the valuation model used.

Given the growing problem of increasingly negative interest rates at the short end of the yield curve and the prospect that this phenomenon is not likely to disappear any time soon, at the very least the adjustments made by the market data providers should give rise to a need for action by all financial institutions. This section focuses on the implications and possible alternative options for IT systems as the area where the institutions' quantitative methods are implemented at industry level.

Trading systems and accounting

Trading and treasury systems usually provide real-time or near real-time yield curves as well as interest rate volatilities for caps, floors and swaptions that are updated at least on a daily basis. They also value any plain vanilla interest-rate options held and at the same time calculate sensitivities for hedging and risk management as well as P&L data. Structured interest rate products also depend on the yield curves and volatilities deposited and are often processed via interest rate structure models integrated into the system. Therefore the operating capability of interest rate derivative trading fundamentally depends on the ability to correctly map the new volatility conventions in the system at short notice.

The correct interpretation of market data is also a basic prerequisite for the determination of fair values for accounting purposes. In many cases, it is the balance-sheet data for interest rate derivatives in particular that are taken over directly from the trading systems.

Shifted lognormal possible with standard tools

Owing to the characteristics outlined above, in practice the shifted lognormal model has a clear advantage over the standard approach: IT systems or software that are capable of valuing vanilla options using Black'76, can also value these derivatives by using the shifted lognormal model. For this purpose, only the forward rate utilised and the strike price of the instrument to be valued must be shifted by an offset of "-Θ" (upwards in case of a negative shift) and the implicit model-adjusted volatility surface must be captured within the system.

In contrast, new formulae must be implemented for the normal model, which presupposes a new release or, to the extent that it is technically feasible, the adjustment of the valuation dynamics.

Parallel shift transfers to forward and swap rates

The following should be noted with regard to the shifted lognormal model: for caplets and floorlets, the underlying is constituted by the relevant forward rate derived from the (LIBOR) yield curve used, while swaptions are constituted by the relevant par swap rate. Instead of having direct access to these rates, the relevant zero rates in the system are generally parameterised (usually after the prior bootstrapping of benchmark interest rates). A parallel shift of the zero rates by "-Θ" only implies an exact parallel shift of the forward rates, provided that the rates are given in accordance with the Continuous Compounding convention. A similar behaviour for swap rates if matching interest periods for both swap legs is assumed while also postponing the fixing for the current period. However, in the past, bank systems often parameterise LIBOR curves using the Annual Compounding convention, where the parallel shift from "Zero-" still translates to the forward rates with a very high degree of approximation (particularly in cases where the interest rates used are close to zero).

Dynamic vs. static shifts

An important consideration when setting up the shift methodology is whether the offsets should be selected per curve, per currency or globally within the system and whether a dynamic adjustment would be provided for. This is closely related to the question of how large a "reserve" should be selected within the shift. If shifted lognormal volatilities are intended to be included into the systems without additional conversion, the shift per curve (or at least currency) supplied by the data provider must be stored and dynamically adjusted. This would entail a simultaneous update of the yield curve (and of the strike prices) of all the instruments to be valued, once the supplied shift has changed. Alternatively, in the case of a statically selected shift and before uploading into the system, the volatilities provided must be converted into a shifted lognormal volatility in line with the offset by using the implied price.

Depending on the flexibility of the system, the shift may be realised via customising features when valuing a financial instrument. This can be achieved by depositing the shift size as an additional user-configured field on the yield curve, by the addition of the shift via API when accessing the curve, or by the manipulation of the strike price in the valuation of the instrument.

However, in many cases, this might require some sort of shadow accounting because of duplicated yield curves and instruments. Conceptually, this must be carefully planned, tested and documented during implementation.

SABR surfaces can still be utilised

A further advantage of the displaced diffusion model is that the – by now widespread – method of parameterisation and interpolation of the volatility skew can be maintained via the SABR model, which is available in many systems (cf. [5]). For this purpose, a displaced SABR surface is in fact used, calibrated to the implicit displaced diffusion volatilities under the shifted forward rates. However, with regard to interest rate levels smaller than 1% and the transition from interpolation to extrapolation, the SABR model displays general weaknesses, dictating the need for relevant analyses (cf. [1]).

In contrast, for normal volatilities, only SABR with β=0 results in a normal interest-rate model with stochastic volatility. Whether or not this approach is useful in practice would need to be examined in detail.

Valuation libraries and structured products

Valuation libraries are predominantly used when structured interest rate products are to be valued using the Hull White or the LIBOR Market Model (LMM). On a case-by-case basis, it needs to be examined whether the relevant model and its implementation are compatible with the approach described above for the shifted lognormal model. In most cases, either the targeted adjustment by the producer is required or a certain degree of fuzziness or theoretical inconsistencies will have to be accepted.

This is exemplified (cf. Lee and Wang, [2]) by the assumption that a flat skew in the lognormal model necessarily implies a declining skew in the shifted lognormal model. It should therefore be noted that the calibration of an adjusted interest rate structure model exclusively in relation to ATM shifted lognormal volatilities is indistinguishable from the retrospective assumption of a certain (non-flat) skew in the "old" model. This effect increases with the size of the shift and must be included in the argument for model assumption purposes.

Risk systems

Market risk systems and their more recent equivalents for counterparty risk (CVA engines) face the same challenges as trading systems from a valuation perspective, as the target values such as value-at risk and exposure are solely based on theoretical valuations.

In addition, any change in model or the associated input market data will always give rise to questions concerning the discontinuity of the histories of market parameters or risk drivers. If the risk drivers for lognormal interest rate volatilities could be parameterised within the system in a meaningful manner (based on yield curve and strike price shifts.), as a rule this should also be possible within a shifted lognormal model.

However, this may pose specific challenges. For example if the yield curves are to be provided both in shifted and original form they must be linked to the same interest risk drivers. If the interest risk drivers themselves are modeled as lognormal, then, as part of the most basic approach the zero rates should act as risk drivers on the shifted yield curve and the original yield curve should be deterministically linked via the shift.

Where this is not feasible due to a lack of flexibility in the system, all interest-based instruments (not only interest-rate options) must be linked to the shifted interest rate, requiring further adjustments to ensure correct valuation. Obviously, the same applies to any valuation software unable to handle negative interest on the yield curve itself or as part of the instrument valuation. However, the latter would constitute a major system adjustment and would therefore need to be addressed within the correct conceptual framework.

Moreover, each shift change may be associated with the necessity of a complete recalibration of the volatility and interest risk factor history in order to correctly assess the volatility of the risk drivers. This can lead to additional problems, particularly if the shift size subsequently decreases, as the recalibration history would then probably no longer be fully usable.

4. Conclusion

The negative interest environment in the market and the ensuing changes to the volatility conventions result in the need for adjustments in almost all financial institutions over the short term. A medium or high necessity for change is required dependent on the instance of application. However, to a large degree, these adjustments can be realised by appropriate parameterisation of a bank's systems and pre-processing during data entry, without need to wait for new software releases.

5. References

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